

# Ginzburg-Landau Expansion in Non-Fermi Liquid Superconductors: Effect of the Mass Renormalization Factor

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## Abstract

We reconsider the Ginzburg-Landau expansion for the case of a non-Fermi liquid superconductor. We obtain analytical results for the Ginzburg-Landau functional in the critical region around the superconducting phase transition,  $T \leq T_c$ , in two special limits of the model, i.e., the spin-charge separation case and the anomalous Fermi liquid case. For both cases, in the presence of a mass renormalization factor, we derived the form and the specific dependence of the coherence length, penetration depth, specific heat jump at the critical point, and the magnetic upper critical field. For both limits the obtained results reduce to the usual BCS results for a two dimensional s-wave superconductor. We compare our results with recent and relevant theoretical work. The results for a d-wave symmetry order parameter do not change qualitatively the results presented in this paper. Only numerical factors appear additionally in our expressions.

## I. INTRODUCTION

The discovery of high temperature superconductivity (HTSC) in 1986 by Bednorz and Müller[1] has caused a lot of enthusiasm among the physics community. Today, after 15 years from their discovery, these materials are still not completely understood from the theoretical point of view. Being a part of the largest family of strongly correlated electron systems, the cuprates present anomalous properties both in the normal and superconducting phase. As a consequence, standard theories, the Landau theory of the Fermi liquid and the BCS theory of the superconducting state, fail to correctly describe the physical properties of these materials. As alternatives, in the case of the normal state, several phenomenological models have been proposed in order to explain their nonmetallic behavior. [2] Despite the fact that the superconducting transition occurs at high temperatures, a characteristic of the ordered phase is the presence of the electron pairs, leading to the idea that a modified BCS theory is appropriate for the description of their superconducting state. We mention that Franz and Tesanovic[3] has put forward a phase fluctuation model for the pseudogap state of cuprate superconductors, which includes a d-wave order symmetry, generating non-Fermi liquid behavior (see their Fig. 1).

In the following, for the description of the normal state we will adopt the model proposed by Anderson[4, 5] which is based on the hypothesis that a two dimensional (2D) system can be described by a Luttinger liquid type theory, similar to the one dimensional (1D) case. The generalization of the Luttinger liquid for the 2D case involves the use of the following Green's function in order to describe the free particles in the normal state of cuprates:

$$G(\vec{k}, i\omega_n) = \frac{g(\alpha)e^{-i\pi\alpha/2}}{\omega_c^\alpha(i\omega_n - \beta\eta\varepsilon_{\vec{k}})^{1/2}(i\omega_n - \beta\varepsilon_{\vec{k}})^{1/2-\alpha}}, \quad (1)$$

where  $\omega_c$  is a cut-off frequency,  $\eta = u_\sigma/u_\rho < 1$  is the ratio of the spin and charge velocities in the system,  $\alpha$  is the non universal exponent related to the anomalous Fermi surface,  $\beta = 2/(\eta+1-2\alpha)$  is the mass renormalization factor, and  $g(\alpha) = \pi\alpha/[2\sin(\pi\alpha/2)]$ . Relations between the different parameters entering Eq. (1) can be obtained by studying different general properties of the Green's function. For example the form of the function  $g(\alpha)$  was obtained by the use of the first sum rule.[6, 7] Based on the formalism proposed by Nolting[8], the necessity of the mass renormalization factor was predicted in Ref. [7] using high order sum rules.

Different choices of the given parameters,  $\alpha$  and  $\eta$ , give us the possibility to distinguish between several different regimes incorporated in the Green's function (Eq. (1)). The most general case is the one of an anomalous spin-charge separation Fermi liquid characterized by  $\eta \neq 1$  and  $\alpha \neq 0$ . Unfortunately, for this general case, analytical results are very difficult to obtain. The spin-charge separation liquid is obtained when  $\eta \neq 1$  and  $\alpha = 0$ , when the Green's function is very similar with the one characterizing the Luttinger liquid. The case of an anomalous Fermi liquid is obtained for  $\eta = 1$  and  $\alpha \neq 0$ . In this case, the spectral function  $A(\vec{k}, \omega) = -ImG(\vec{k}, \omega)/\pi$  satisfies the homogeneity relation  $A(\Lambda\vec{k}, \Lambda\omega) = \Lambda^{-1+\alpha}A(\vec{k}, \omega)$  with an exponent  $\alpha > 0$ . As a general feature the usual Fermi liquid limit is obtained by simply considering  $\eta = 1$  and  $\alpha = 0$ .

Previously, this model was used in order to investigate the superconducting state in non-Fermi liquid systems.[9, 10, 11, 12] Several properties of the superconducting state were investigated assuming the validity of the Gorkov's equations, with the usual normal state Fermi liquid Green's function replaced by the non-Fermi liquid one given by Eq. (1). For the case of the anomalous Fermi liquid ( $\alpha \neq 0$ ,  $\eta = 1$ ) there have been previous analysis of the Ginzburg-Landau parameters in the framework of fluctuation theory[13] and general Ginzburg-Landau functional theory[14], but these two approaches missed the role of the mass renormalization factor, leading to uncomplete results.

In this paper, based on the Ginzburg-Landau functional formalism, we investigate both the spin-charge separation liquid and the anomalous Fermi liquid in the superconducting state and we evaluate the coherence length, the penetration depth, the specific heat jump at the critical point, and the magnetic upper critical field.

## II. GENERAL FORMALISM OF THE GINZBURG-LANDAU FUNCTIONAL

Let us first consider the general case of a  $s$ -wave superconductor. In the standard Ginzburg-Landau expansion, the difference between the superconducting and normal state free energy can be written as:

$$F_S - F_N = A|\Delta_{\mathbf{q}}|^2 + q^2C|\Delta_{\mathbf{q}}|^2 + \frac{B}{2}|\Delta_{\mathbf{q}}|^4, \quad (2)$$

where  $S$  denotes the superconducting state,  $N$  the normal state,  $\Delta_{\mathbf{q}}$  is the Fourier transform of the order parameter, and  $A$ ,  $B$ ,  $C$  are the Ginzburg-Landau coefficients. Following Ref.

[15] we can express these coefficients as:

$$A \equiv \frac{1}{V} - T \sum_n \int \frac{d^2k}{(2\pi)^2} \left[ \mathcal{G}(\vec{k}, i\omega_n) \mathcal{G}(-\vec{k}, -i\omega_n) + \mathcal{F}(\vec{k}, i\omega_n) \mathcal{F}(-\vec{k}, -i\omega_n) \right], \quad (3)$$

$$B \equiv T_c \sum_n \int \frac{d^2k}{(2\pi)^2} \left[ \mathcal{G}(\vec{k}, i\omega_n) \mathcal{G}(-\vec{k}, -i\omega_n) + \mathcal{F}(\vec{k}, i\omega_n) \mathcal{F}(-\vec{k}, -i\omega_n) \right]^2, \quad (4)$$

and  $C$  as the coefficient of the  $q^2|\Delta_{\mathbf{q}}|^2$  term in the Taylor expansion of the following term:

$$-T_c \sum_n \int \frac{d^2k}{(2\pi)^2} \left[ \mathcal{G}(\vec{k} + \frac{\vec{q}}{2}, i\omega_n) \mathcal{G}(-\vec{k} + \frac{\vec{q}}{2}, -i\omega_n) + \mathcal{F}(\vec{k} + \frac{\vec{q}}{2}, i\omega_n) \mathcal{F}(-\vec{k} + \frac{\vec{q}}{2}, -i\omega_n) \right]. \quad (5)$$

For the case of a  $s$ -wave superconductor  $\mathcal{G}(\vec{k}, i\omega_n)$  and  $\mathcal{F}(\vec{k}, i\omega_n)$  represent the normal and anomalous Green's functions, respectively, in the superconducting state.[16] In Eq. (3),  $V$  represents the absolute value of the attractive interaction leading to the superconducting phase transition. In general, in order to evaluate  $A$ , the interaction potential is replaced using the critical temperature equation, leading for the  $s$ -wave case to the following value:

$$A_0 = N_0 \frac{T - T_{c0}}{T_{c0}}, \quad (6)$$

where  $T_{c0}$  stays for the standard BCS critical temperature and  $N_0$  the free electron gas density of states at the Fermi level. The other two coefficients,  $B$  and  $C$ , can be calculated after some simple but tedious algebra. The results are:

$$B_0 = \frac{7\zeta(3)N_0}{8\pi^2 T_{c0}^2}, \quad (7)$$

and

$$C_0 = \frac{7\zeta(3)v_F^2 N_0 I(\theta)}{16\pi^2 T_{c0}^2} \quad (8)$$

where  $\zeta(z)$  is the Riemann function,  $v_F$  is the Fermi velocity and

$$I(\theta) = \begin{cases} \frac{1}{2}, & \text{2D case} \\ \frac{1}{3}, & \text{3D case} \end{cases}. \quad (9)$$

Of course for the correct dimensional evaluation of the Ginzburg-Landau coefficients, the density of states  $N_0$  should be considered according to the dimensionality. As we expected  $A = 0$  gives the mean field critical temperature of the superconducting phase transition, whereas  $B$  and  $C$  are weakly temperature dependent and are evaluated at  $T = T_{c0}$ .

Standard BCS theory considered a simple  $s$ -wave symmetry of the order parameter in standard superconductors. However, at the present time is generally accepted that in HTSC the order parameter symmetry is more of  $d$ -wave type.[17] Accordingly, an evaluation of the Ginzburg-Landau coefficients for HTSC should consider a momenta dependent order parameter  $\Delta(\vec{k}) = \Delta\psi(\vec{k})$ , where  $\psi(\vec{k})$  is a coefficient which includes the symmetry factor. For  $s$ -wave superconductors this is simple  $\psi(\vec{k}) = 1$ . For the 2D  $d$ -wave symmetry case,  $\psi(\vec{k}) = \cos(2\theta_{\vec{k}})$ , where  $\theta_{\vec{k}} = \arctan(k_y/k_x)$ . A calculation of the Ginzburg-Landau coefficients in the framework of  $d$ -wave symmetry is straightforward leading to the following results

$$A_0^{(d)} = K_A A_0, \quad (10)$$

$$B_0^{(d)} = K_B B_0, \quad (11)$$

$$C_0^{(d)} = K_C C_0, \quad (12)$$

where  $K_A = K_C = 1/2$  and  $K_B = 3/8$ .

As it is well known, the knowledge of the Ginzburg-Landau coefficients give direct insight on several of the superconducting state properties. Among these properties, we notice two characteristic lengths, namely the coherence length and the penetration depth of the magnetic field. In the following, we will present the standard BCS results for a 2D  $s$ -wave superconductor. The coherence length at a specific temperature,  $\xi_{BCS}(T)$ , characterizes the size of the Cooper pair and can be expressed as:

$$\xi_{BCS}(T) = \sqrt{-\frac{C_0}{A_0}} = 0.74 \frac{\xi_{BCS}(0)}{\sqrt{1 - T/T_{c0}}}, \quad (13)$$

where  $\xi_{BCS}(0) = 0.18v_F/T_{c0}$  represents the coherence length at  $T = 0$ . The other characteristic length is the London penetration depth of a magnetic field in the superconductor, and for the 2D  $s$ -wave BCS case is expressed as:

$$\lambda_{BCS}(T) = \sqrt{-\frac{c^2}{32\pi e^2} \frac{B_0}{A_0 C_0}} = \frac{1}{\sqrt{2}} \frac{\lambda_{BCS}(0)}{\sqrt{1 - T/T_{c0}}}, \quad (14)$$

with  $\lambda_{BCS}^2(0) = mc^2/(4\pi ne^2)$  being the London penetration depth at  $T = 0$  ( $c$  stays for the usual value of the light speed in vacuum,  $n$  is the density, and  $e$  represents the electron charge).

Other two important quantities which emerge directly from the Ginzburg-Landau coefficients are the specific heat jump at the critical point and the upper magnetic critical field  $H_{c2}$ . In terms of these coefficients, the specific heat jump at the critical point can be expressed as:

$$\frac{\Delta C_v^{BCS}}{\Omega} = \frac{T_{c0}}{B_0} \left( \frac{A_0}{T - T_{c0}} \right)^2, \quad (15)$$

with  $\Omega$  being the sample volume. Based on this equation at the critical point one obtain:

$$\left( \frac{\Delta C_v^{BCS}}{\Omega} \right)_{T_{c0}} = N_0 \frac{8\pi^2 T_{c0}}{7\zeta(3)}. \quad (16)$$

The upper magnetic critical field is given by:

$$H_{c2}^{BCS} = -\frac{\phi_0}{2\pi} \frac{A_0}{C_0}, \quad (17)$$

$\phi_0 = \pi c/e$  being the quantum of the magnetic flux. The slope of the curve for the upper critical field can be easily obtained from Eq. (17) as:

$$\left| \frac{dH_{c2}^{BCS}}{dT} \right|_{T_{c0}} = \frac{16\pi\phi_0}{7\zeta(3)v_F^2} T_{c0}. \quad (18)$$

Beside these important parameters for the superconducting state, based on the superconducting order parameter fluctuations, some important features of the normal state were investigated. Among different properties, the presence of electron pairs in the critical region above  $T_c$  is responsible for the presence of a gap in the excitation spectrum of the normal state quasiparticles.[18, 19] For the case of high temperature superconductors this behavior is expected to be stronger than in standard metallic systems due to a quasi-two-dimensional structure which is responsible for an enhanced critical region around the superconducting phase transition. This property was used to explain recent ARPES and tunnelling measurements in cuprates which experimentally prove the existence of the pseudogap state in their normal phase.[13, 20]

### III. ANALYTICAL RESULTS IN THE NON-FERMI LIQUID CASE

In this section, we will apply the Ginzburg-Landau functional formalism to the case of a  $s$ -wave non-Fermi liquid superconductor. Our evaluation of normal and anomalous Green's

functions in the superconducting state starts with the hypothesis that standard Gorkov's equations are still valid. Working on the same hypothesis Muthukumar et al. [11], following Gorkov's classic procedure,[16] derived the Ginzburg-Landau equation and obtained the Ginzburg-Landau coefficients,  $A$  and  $B$ . Based on these calculations the value of the upper critical field,  $H_c(T)$  near  $T_c$  was obtained for the case of the anomalous Fermi liquid. However, in our analysis we will extend the calculation to the evaluation of the other Ginzburg-Landau coefficient, namely  $C$ , and we will obtain analytical results for the superconducting state parameters in both spin-charge separation and anomalous Fermi liquids cases. The role of the mass renormalization factor,  $\beta$ , is particularly discussed for the anomalous Fermi liquid case, situation in which a previous estimation of the Ginzburg-Landau coefficients was made by Moca.[14]

### A. The spin-charge separation liquid

Here, we will focus our attention on the spin-charge separation liquid case, denoted by the following choice of the input parameters in Eq. (1), namely,  $\alpha = 0$  and  $\eta \neq 1$ . The corresponding form of the Green's function is:

$$G(\vec{k}, i\omega_n) = \frac{1}{(i\omega_n - \eta\beta\varepsilon_{\vec{k}})^{1/2}(i\omega_n - \beta\varepsilon_{\vec{k}})^{1/2}}, \quad (19)$$

with the mass renormalization factor given by  $\beta = 2/(1 + \eta)$ . It is simple to see that for  $\eta \rightarrow 1$  the usual form for the Fermi liquid is recovered.

The critical temperature,  $T_c(\eta)$ , on this model was first calculated by Sudbo[12]. A more careful analysis of this temperature, which also includes the mass renormalization factor, leads to the following value:

$$T_c(\eta) = \frac{2\gamma_E}{\pi} \frac{2\omega_D}{1 + \eta} \exp \left[ -\frac{\pi}{(1 + \eta)K(\sqrt{1 - \eta^2})} \frac{1}{N_0 V} \right], \quad (20)$$

where  $\gamma_E$  is the Euler constant,  $\omega_D$  is the Debye frequency, and  $K(k)$  is the complete elliptic integral of the first kind.

A simple but laborious calculation of the Ginzburg-Landau parameters leads to the following values:

$$A(\eta) = N_0 \frac{T - T_c(\eta)}{T_c(\eta)} f_A(\eta), \quad (21)$$

$$B(\eta) = \frac{7\zeta(3)N_0}{8\pi^2 T_c^2(\eta)} f_B(\eta), \quad (22)$$

$$C(\eta) = \frac{7\zeta(3)N_0v_F^2}{32\pi^2T_c^2(\eta)}f_C(\eta) , \quad (23)$$

where we introduced the following notations

$$f_A(\eta) = \frac{(1+\eta)K(\sqrt{1-\eta^2})}{\pi} , \quad (24)$$

$$f_B(\eta) = \frac{1+\eta}{2}F\left(1, \frac{1}{2}; 2; 1-\eta^2\right) , \quad (25)$$

$$\begin{aligned} f_C(\eta) = & \frac{1}{2(1+\eta)} \left\{ \frac{3}{2} \left[ \frac{3}{2}F\left(\frac{1}{2}, \frac{1}{2}; 3; 1-\eta^2\right) \right. \right. \\ & + \eta F\left(\frac{3}{2}, \frac{1}{2}; 3; 1-\eta^2\right) + \frac{3\eta^2}{2}F\left(\frac{5}{2}, \frac{1}{2}; 3; 1-\eta^2\right) \Big] \\ & \left. - F\left(\frac{1}{2}, \frac{1}{2}; 2; 1-\eta^2\right) - \eta^2 F\left(\frac{3}{2}, \frac{1}{2}; 2; 1-\eta^2\right) \right\} , \end{aligned} \quad (26)$$

$F(\alpha, \beta; \gamma; z)$  being the hypergeometric function. Making the limit  $\eta \rightarrow 1$  the standard BCS results are recovered.

With all three Ginzburg-Landau coefficients, we can obtain the corresponding quantities for the superconducting state. The coherence length can be written as:

$$\frac{\xi(\eta, T)}{\xi_{BCS}(T)} = \frac{1}{f_T(\eta)} \sqrt{\frac{f_C(\eta)}{f_A(\eta)}} \sqrt{\frac{1 - T/T_{c0}}{1 - T/[f_T(\eta)T_{c0}]}} , \quad (27)$$

with

$$\begin{aligned} f_T(\eta) &= \frac{T_c(\eta)}{T_{c0}} \\ &= \frac{2}{1+\eta} \exp \left[ \left( 1 - \frac{\pi}{(1+\eta)K(\sqrt{1-\eta^2})} \right) \frac{1}{N_0V} \right] . \end{aligned} \quad (28)$$

As we expect  $f_T(\eta = 1) \rightarrow 1$ . In Fig.1 we plot the  $\eta$ -dependence of the ratio between the coherence length in the spin-charge separation liquid and the standard BCS case. We observe that, for  $\eta \neq 1$ , the value of the coherence length is lower than the one in the standard BCS case. The considered values of the  $T/T_{c0}$  are justified by the range of the critical region around the transition temperature.

The London penetration depth is obtained as:

$$\frac{\lambda(\eta, T)}{\lambda_{BCS}(T)} = \sqrt{\frac{f_B(\eta)}{f_A(\eta)f_C(\eta)}} \sqrt{\frac{1 - T/T_{c0}}{1 - T/[f_T(\eta)T_{c0}]}} . \quad (29)$$

As in the BCS case, the temperature dependence of London penetration depth and coherence length are the same. In Fig.2, we plot the  $\eta$ -dependence of the ratio between the penetration

depth corresponding to the spin-charge separation liquid and the standard BCS case. As the separation parameter decreases a lower value for the penetration depth is obtained.

The value of the specific heat jump at the critical point for  $\eta \neq 1$  was first calculated in Ref. [21] based on the Pauli theorem. Using the Ginzburg-Landau coefficients, one finds:

$$\frac{\Delta C_v(\eta)}{T_c(\eta)} \frac{T_{c0}}{\Delta C_v^{BCS}} = \frac{f_A^2(\eta)}{f_B(\eta)}, \quad (30)$$

a value which differ from the previous one obtained in Ref. [21] by the inclusion of the mass renormalization factor. In Fig.3, we plot the  $\eta$ -dependence of the specific heat jump at the critical point renormalized by the same ratio considered in the standard BCS case. One can see that higher values of the specific heat jump at the transition point can be expected as the spin-charge separation parameter  $\eta$  decreases.

Finally, we are going to evaluate the upper critical magnetic field,  $H_{c2}$ . One finds:

$$\frac{H_{c2}(\eta)}{H_{c2}^{BCS}} = \frac{f_A(\eta)f_T^2(\eta)}{f_C(\eta)} \frac{1 - T/[f_T(\eta)T_{c0}]}{1 - T/T_{c0}}, \quad (31)$$

leading to a change on the slope of the curve for the upper critical field near the transition temperature:

$$h(\eta) = \frac{\left| \frac{dH_{c2}}{dT} \right|_{T=T_c(\eta)}}{\left| \frac{dH_{c2}^{BCS}}{dT} \right|_{T=T_{c0}}} = \frac{f_A(\eta)f_T(\eta)}{f_C(\eta)}. \quad (32)$$

In Fig.4, we plot the relative value of the slope of the curve for the upper critical field as function of the spin-charge separation parameter. An increment of this slope is predicted as the value of  $\eta$  decreases.

At this particular moment we would like to discuss the presence of a fluctuating vector field,  $\vec{a}$ , in addition to the electromagnetic vector field,  $\vec{A}$  as it has been discussed in the literature.[22, 23, 24, 25] We argue that the full superconducting order parameter,  $\Delta$ , calculated in this paper, for the spin-charge separation case, becomes

$$\Delta^2 \approx \frac{1}{1 + f(\eta)} \left[ \Delta^{(s)2} + f(\eta)\Delta^{(h)2} \right], \quad (33)$$

where  $f(\eta) \rightarrow 1$  for  $\eta \rightarrow 1$  and  $\Delta^{s,h}$  refers to the order parameter for the spinons (holons), respectively. Along with the assumption of Eq. (33) is implicit that both order parameters are small. In Eq. (33),  $f(\eta)$  is fixed by the ration between the two critical temperatures, namely,  $T_c^{(s,h)}$ . Then, with these assumptions, we find that our Ginzburg-Landau function

can be expressed as

$$\begin{aligned}
F_S - F_N = & A_s |\Delta_{\mathbf{q}}^{(s)}|^2 + q^2 C_s |\Delta_{\mathbf{q}}^{(s)}|^2 + \frac{B_s}{2} |\Delta_{\mathbf{q}}^{(s)}|^4 \\
& + A_h |\Delta_{\mathbf{q}}^{(h)}|^2 + q^2 C_h |\Delta_{\mathbf{q}}^{(h)}|^2 + \frac{B_h}{2} |\Delta_{\mathbf{q}}^{(h)}|^4 \\
& + B_{s,h} |\Delta^{(s)}|^2 \times |\Delta^{(h)}|^2 .
\end{aligned} \tag{34}$$

In Eq. (34), for example,  $A_s = A/(1 + f(\eta))$ , etc. Transforming our  $G - L$  functional to real space and substituting the vector potentials, we obtain

$$\begin{aligned}
F_S - F_N = & A_s |\Delta^{(s)}|^2 + C_s |(-i\nabla - 2\vec{a}^{(s)})\Delta^{(s)}|^2 + \frac{B_s}{2} |\Delta^{(s)}|^4 \\
& + B_{s,h} |\Delta^{(s)}|^2 \times |\Delta^{(h)}|^2 \\
& + A_h |\Delta^{(h)}|^2 + C_h |(-i\nabla - \frac{e}{\hbar c} \vec{A}^{(e)} - \vec{a}^{(h)})\Delta^{(h)}|^2 \\
& + \frac{B_h}{2} |\Delta^{(h)}|^4 + \frac{1}{8\pi} (\nabla \times \vec{A}^{(e)})^2 + f_{gauge} ,
\end{aligned} \tag{35}$$

where  $f_{gauge}$  is given by

$$f_{gauge} \equiv \frac{\varrho}{2} (\nabla \times \vec{a})^2 . \tag{36}$$

In Eq. (35),  $\vec{a}$  is the fluctuating vector potential (internal gauge field).[23] The factor of 2 in front of  $\vec{a}$  in the spinon gradient term reflects the fact that pairs of spinons are assumed to condensate.  $f_{gauge}$  describes the dynamics of the internal gauge field,  $\vec{a}$ . The internal gauge field,  $\vec{a}$ , serves only to enforce the local constraint  $b_i^\dagger b_i + f_{i,\sigma}^\dagger f_{i,\sigma} = 1$ . The contribution  $f_{gauge}$  has been justified by Sachdev[26] and Nagaosa-Lee.[23] Franz and Tesanovich argue that  $\varrho$  should be zero in order to reproduce the experimental data. Eq. (35) is also similar to Eq. (2) of Franz and Tesanovic.[23] We recover the results of Eq. (2) of Ref. [23]: we do have a contribution of the type  $\propto |\Delta^{(s)}|^2 \times |\Delta^{(h)}|^2$ , where the coefficient  $B_{s,h} = 2B f_1 f_2$ , where  $f_1 = 1/(1 + f(\eta))^2$  and  $f_2 = f^2(\eta) f_1^2$ .

There is another interpretation due to Muthukumar, Weng and Sheng.[22] They have two fluctuating gauge fields, one due to holons and another due to spinons.

## B. The anomalous Fermi liquid

The anomalous Fermi liquid (Eq. (1)) is defined in the limit  $\eta = 1$  and  $\alpha \neq 0$ , which implies that the normal state Green's function can be written as:

$$G(\vec{k}, i\omega_n) = \frac{g(\alpha)e^{-i\pi\alpha/2}}{\omega_c^\alpha(i\omega_n - \beta\varepsilon_{\vec{k}})^{1-\alpha}}, \quad (37)$$

with the mass renormalization factor given by  $\beta = 1/(1 - \alpha)$ . As we expected, for  $\alpha \rightarrow 0$ , the standard Fermi liquid theory is recovered.

A superconducting phase transition in this system occurs at a critical temperature:[7]

$$T_c^{2\alpha}(\alpha) = \frac{1}{M(\alpha)} \left[ N(\alpha) \left( \frac{\omega_D}{1 - \alpha} \right)^{2\alpha} - \frac{1}{(1 - \alpha)g^2(\alpha)} \frac{\omega_c^{2\alpha}}{P(\alpha)N_0V} \right], \quad (38)$$

only if the value of the attractive interaction is higher than a certain critical value,[7]  $V > V_{cr}$ . The constants entering Eq. (38) are given by  $P(\alpha) = 2^{2\alpha} \sin[\pi(1 - \alpha)]/\pi$ ,  $M(\alpha) = \Gamma^2(\alpha)[1 - 2^{1-2\alpha}]\zeta(2\alpha)$ , and  $N(\alpha) = \Gamma(1 - 2\alpha)\Gamma(\alpha)/[2\alpha\Gamma(1 - \alpha)]$ ,  $\Gamma(x)$  being the gamma function. Despite the fact that the value of the critical temperature is much more complicated in this case, the standard BCS value still can be obtained as  $\alpha \rightarrow 0$ . [9, 10]

The first attempt to calculate the Ginzburg–Landau coefficients for the case of the anomalous Fermi liquid was made by Moca,[14] but our analysis is justified by the necessity of the mass renormalization factor (omitted in Ref. [14]) and by some misprints in the reported results. However, we also estimate the value of the magnetic upper critical field,  $H_{c2}$ .

Following the same procedure applied previously to the spin-charge separation liquid, one finds that the Ginzburg–Landau parameters are expressed as:

$$A(\alpha) = N_0 \frac{T - T_c(\alpha)}{T_c(\alpha)} f_A(\alpha), \quad (39)$$

$$B(\alpha) = \frac{7\zeta(3)N_0}{8\pi^2 T_c^2(\alpha)} f_B(\alpha), \quad (40)$$

$$C(\alpha) = \frac{7\zeta(3)N_0 v_F^2}{32\pi^2 T_c^2(\alpha)} f_C(\alpha), \quad (41)$$

where we introduced the following functions:

$$f_A(\alpha) =$$

$$\begin{aligned}
& 2\alpha(1-\alpha)g^2(\alpha)P(\alpha)M(\alpha) \left[ \frac{N(\alpha)}{M(\alpha)} \right] \left[ \frac{\omega_D}{(1-\alpha)\omega_c} \right]^{2\alpha} \\
& \times \left[ 1 - \frac{1}{N_0V} \frac{g^{-2}(\alpha)}{(1-\alpha)P(\alpha)N(\alpha)} \left( \frac{(1-\alpha)\omega_c}{\omega_D} \right)^{2\alpha} \right],
\end{aligned} \tag{42}$$

$$\begin{aligned}
f_B(\alpha) &= \frac{2(1-\alpha)B\left(\frac{1}{2}, \frac{3}{2} - \alpha\right)}{\pi} \frac{2^{3-4\alpha} - 1}{7} \frac{\zeta(3-4\alpha)}{\zeta(3)} \\
& \times g^4(\alpha) \left( \frac{2\pi\omega_D}{(1-\alpha)\omega_c} \right)^{4\alpha} \left[ \frac{N(\alpha)}{M(\alpha)} \right]^2 \\
& \times \left[ 1 - \frac{1}{N_0V} \frac{g^{-2}(\alpha)}{(1-\alpha)P(\alpha)N(\alpha)} \left( \frac{(1-\alpha)\omega_c}{\omega_D} \right)^{2\alpha} \right]^2,
\end{aligned} \tag{43}$$

$$\begin{aligned}
f_C(\alpha) &= \frac{g^2(\alpha) \cos[\pi\alpha]}{1-\alpha} \\
& \times \frac{2^{3-2\alpha} - 1}{7} \frac{\zeta(3-2\alpha)}{\zeta(3)} \frac{N(\alpha)}{M(\alpha)} \left[ \frac{2\pi\omega_D}{(1-\alpha)\omega_c} \right]^{2\alpha} \\
& \times \frac{2(1-\alpha)(2-\alpha)B\left(\frac{1}{2}, \frac{5}{2} - \alpha\right) - (1-\alpha)B\left(\frac{1}{2}, \frac{3}{2} - \alpha\right)}{\pi} \\
& \times \left[ 1 - \frac{1}{N_0V} \frac{g^{-2}(\alpha)}{(1-\alpha)P(\alpha)N(\alpha)} \left( \frac{(1-\alpha)\omega_c}{\omega_D} \right)^{2\alpha} \right],
\end{aligned} \tag{44}$$

where  $B(x, y)$  represents the beta function. Note that the first two coefficients, despite the mass renormalization factor, are the same as the ones reported in Ref. [14], whereas the last one differs from the one already reported. The superconducting state properties can be evaluated by introducing a new function,  $f_T(\alpha)$ , defined as the ratio of the critical temperature in the anomalous Fermi liquid and the one corresponding to the standard BCS case:

$$\begin{aligned}
f_T(\alpha) &= \frac{\pi}{2\gamma_E(1-\alpha)} \left[ \frac{N(\alpha)}{M(\alpha)} \right]^{1/2\alpha} \times \\
& \exp \left\{ \left[ 1 - \frac{1}{N_0V} \frac{g^{-2}(\alpha)}{(1-\alpha)P(\alpha)N(\alpha)} \left( \frac{(1-\alpha)\omega_c}{\omega_D} \right)^{2\alpha} \right]^{1/2} \right\}.
\end{aligned} \tag{45}$$

In this expression we made use of the exponential form of the critical temperature reported by Grosu et al.[10] To express the superconducting state properties one can make use of the

previous expression obtained for the spin-charge separation liquid with the specification that all the functions  $f_i(\eta)$  should be replaced by their correspondent in the anomalous Fermi liquid  $f_i(\alpha)$  ( $i = A, B, C, T$ ).

In Fig.5, we plot the coherence length for the anomalous Fermi liquid related to the standard BCS value as function of the non-Fermi parameter  $\alpha$  for different temperatures in the critical region. The inclusion of the mass renormalization factor  $\gamma = 1/(1 - a)$  changes completely the slope of the curve with respect to previous reported results[14], the coherence length decreasing as  $\alpha$  increases.

Fig.6, shows the ratio of the penetration depth corresponding to the anomalous Fermi liquid with respect to the standard BCS value as function of the non-Fermi parameter  $\alpha$ . For most of the interval a decreasing of the penetration depth as function of the non-Fermi parameter is obtained. The divergence obtained as  $\alpha$  approaches the limit value  $\alpha \rightarrow 0.5$  could be a simple effect related to the divergence of several quantities involved in the calculation. A similar effect was observed also in the critical temperature dependence on the non-Fermi parameter  $\alpha$ . [9, 10, 11]

The dependence of the specific heat jump at the transition temperature is plot in Fig.7. Our results show that smaller values than in the standard BCS case can be achieved for  $0 < \alpha \leq 0.3$ . This conclusion was also obtained in Ref. [14]. However, some changes due to the inclusion of the mass renormalization factor can be seen in our plot.

A first discussion on the magnetic upper critical field was made by Muthukumar et al.[11] as function of the non-Fermi parameter  $\alpha$ . However, due to the incorrect form of the Green's function used in their calculation a qualitatively different dependence of the upper critical field as function of the non-Fermi parameter  $\alpha$  is expected. In Fig.8, we plot the slope of the magnetic upper critical field as function of the non-Fermi parameter  $\alpha$ , showing that an increment of this parameter occurs as  $\alpha$  increases.

#### IV. CONCLUSIONS AND DISCUSSIONS

In this paper, we have discussed the superconducting state properties in systems described by a non-Fermi liquid normal state. We specifically analyzed two limit cases corresponding to the spin-charge separation and anomalous Fermi liquid cases. Our analysis is based on the evaluation of the Ginzburg-Landau coefficients corresponding to the superconducting

phase transition, which are used to evaluate the coherence length, the London penetration depth, the specific heat jump and the magnetic upper critical field. By comparing our results with experimental data on high temperature superconductors we could fix the two parameters,  $\eta$  and  $\alpha$ , entering the model. For the case of high temperature superconductors, the experimental data should be considered in the underdoped region of the phase diagram, where it is well known that the non-Fermi character of the system is much stronger than in the overdoped regime.

In general, some similarities can be observed in both limits of the non-Fermi liquid description. Let us turn our attention on the coherence length. It is well known that in high temperature superconductors, the value of this parameter is much smaller than the one corresponding to standard BCS superconductors.[27] We can see from Figs.1 and 5 that such a behavior can be achieved when the spin-charge separation parameter  $\eta$  decreases, or as the non-Fermi parameter  $\alpha$  increases. Basically both situations have to do with a stronger non-Fermi character of the system. The fact that the coherence length decreases to small values can be interpreted also in terms of a crossover problem, where in place of varying the attractive interaction leading to the formation of the Cooper pair, one can vary the non-Fermi parameter of the problem with the same result, which clearly identifies the fact that the non-Fermi character of the systems is done by the interaction between the component particles.

In both cases, the standard Fermi liquid can be obtained by setting  $\eta = 1$  or  $\alpha = 0$ . For the anomalous Fermi liquid case a direct comparison of the specific heat data with experimental results for high temperature superconductors[28] shows that  $\alpha$  should satisfy the condition  $0.2 < \alpha < 0.4$ . This is because the specific heat jump at the critical temperature has smaller values than in the BCS case. Such values for the non-Fermi parameter  $\alpha$  agree with the observed experimental data for the penetration depth.[29] One can see in this case that a strange divergence of different physical quantities occurs once the value  $\alpha \rightarrow 0.5$  is approached. This unphysical result is a consequence of the various mathematical approximations which we used on the calculation of these parameters. However, direct comparisons with the experimental data exclude this value as a possible correct value for the non-Fermi parameter  $\alpha$ . A similar analysis can be done also for the spin-charge separation liquid.

In short, we have to note that despite the good agreement between our theoretical results and the experimental data one cannot conclude that an extension of the Luttinger liquid

theory in two dimensions is the correct answer for the high temperature superconductivity, at least as long as a direct microscopic theory can not prove the validity of the phenomenological Green's function used in the model.

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- [1] G. Bednorz and K.A. Müller, *Z. Phys. B* **64**, 189 (1986).
  - [2] See, for example, P. Brusov, *Mechanisms of High Temperature Superconductivity* (Rostov State University Publishing, 1999).
  - [3] M. Franz and Z. Tesanovic, *Phys. Rev. Lett.* **87**, 257003 (2001).
  - [4] P.W. Anderson, *The Theory of Superconductivity in High- $T_c$  Cuprates*. (Princeton, 1997).
  - [5] P.W. Anderson, *Science* **256**, 1526 (1992).
  - [6] L. Yin and S. Chakravarty, *Int. J. Mod. Phys. B* **10**, 805 (1996).
  - [7] J.J. Rodríguez-Núñez, I. Țifrea, and S.G. Magalhães, *Phys. Rev. B* **62**, 4026 (2000).
  - [8] W. Nolting, *Z. Phys.* **255**, 25 (1972); H. Herrmann and W. Nolting, *J. Magn. Magn. Mat.* **170**, 253 (1997).
  - [9] I. Țifrea, Ph. Thesis, University of Cluj, 1998 (unpublished).
  - [10] I. Grosu, I. Țifrea, M. Crisan, and S. Yoksan, *Phys. Rev. B* **56**, 8298 (1997).
  - [11] V.N. Muthukumar, D. Sa and M. Sardar, *Phys. Rev. B* **52**, 9647 (1995).
  - [12] A. Sudbo, *Phys. Rev. Lett.* **74**, 2575 (1995).
  - [13] M. Crisan, C.P. Moca, and I. Țifrea, *Phys. Rev. B* **59**, 14680 (1999).
  - [14] C.P. Moca, *Physica C* **349**, 113 (2001).
  - [15] A.I. Posazhennikova and M.V. Sadovskii, *Zh. Eksp. Theor. Fiz* **115**, 632 (1999) [*JETP* **88**,

- 347 (1999)].
- [16] See for example, A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, 1971).
  - [17] C.C. Tsuei and J.R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).
  - [18] A. Schmid, Z. Physik **231**, 324 (1970).
  - [19] S. Takada, Prog. Theor. Phys. **44**, 298 (1970).
  - [20] Y.M. Vilk and A.-M.S. Tremblay, J. Phys. I **7**, 1309 (1997).
  - [21] I. Țifrea and M. Crisan, Eur. Phys. J. B **4**, 175 (1998).
  - [22] V. N. Muthukumar, Z. Y. Weng and D. N. Sheng, cond-mat/0106225; ibidem, cond-mat/0112339; Z. Y. Weng and V. N. Muthukumar, cond-mat/0202079.
  - [23] M. Franz and Z. Tesanovic, Phys. Rev. B **63**, 064516 (2001); P. A. Lee and N. Nagaosa, Phys. Rev. B **46**, 5621 (1992).
  - [24] Z. Y. Weng, D. N. Sheng, and C. S. Ting, Phys. Rev. B **61**, 12328 (2000-II); ibidem, Phys. Rev. Lett. **80**, 5401 (1998).
  - [25] Z. Y. Weng, V. N. Muthukumar, D. N. Sheng, and C. S. Ting, Phys. Rev. B **63**, 075102 (2001).
  - [26] S. Sachdev, Phys. Rev. B **45**, 389 (1992).
  - [27] T.M. Riseman, J.H. Brewer, K.H. Chow, W.N. Hardy, R.F. Kiefl, S.R. Kreitzman, R. Liang, W.A. MacFarlane, P. Mendels, G.D. Morris, J. Rammer, J.W. Schneider, C. Niedermayer, and S.L. Lee, Phys. Rev. B **52**, 10569 (1995).
  - [28] J.L. Tallon and J.W. Loram, Physica C **349**, 53 (2001).
  - [29] C. Panagopoulos, J.R. Cooper, and T. Xiang, Phys. Rev. B **57**, 13 422 (1998).

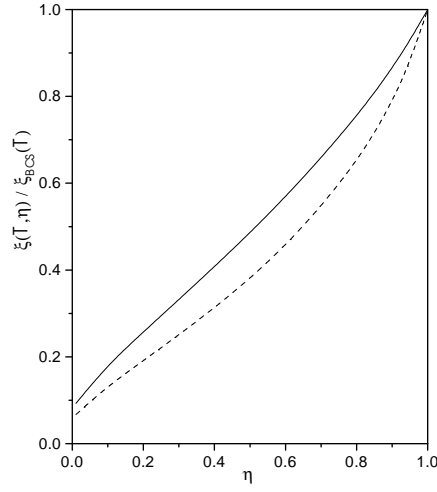


FIG. 1: The coherence length ratio as function of the spin-charge separation parameter  $\eta$  for different values of the  $T/T_{c0}$  ratio. The full line correspond to a value  $T/T_{c0} = 0.8$ , and the dashed line to  $T/T_{c0} = 0.9$ .

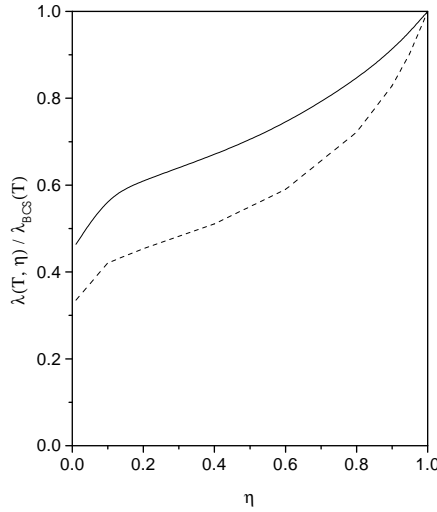


FIG. 2: The penetration depth ratio as function of the spin-charge separation parameter  $\eta$  for different values of the  $T/T_{c0}$  ratio. The full line correspond to a value  $T/T_{c0} = 0.8$ , and the dashed line to  $T/T_{c0} = 0.9$ .

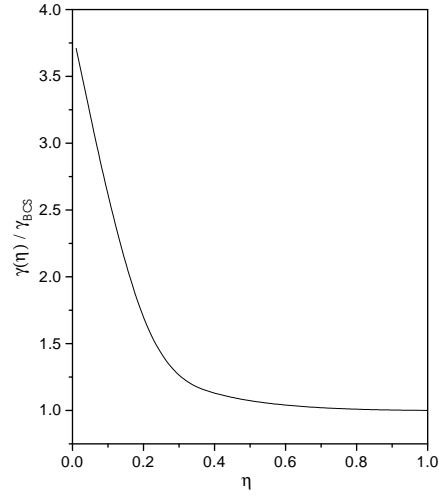


FIG. 3: The specific heat jump at the critical point ratio as function of the spin-charge separation parameter  $\eta$ .

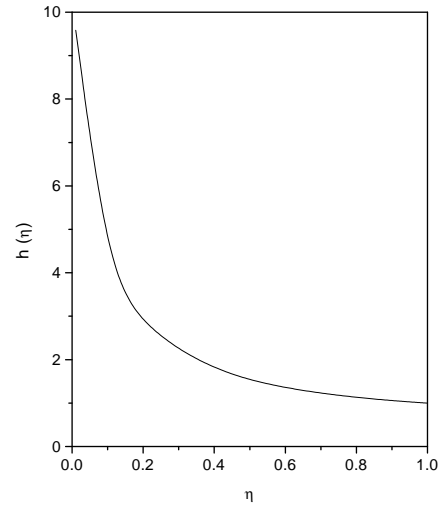


FIG. 4: The relative slope of the curve for the magnetic upper critical field as function of the spin-charge separation parameter  $\eta$ .

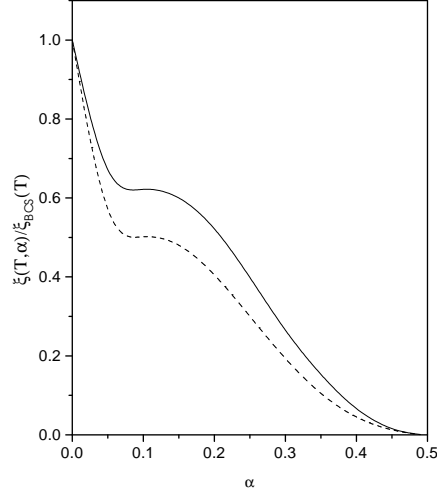


FIG. 5: The coherence length ratio as function of the non-Fermi parameter  $\alpha$  for different values of the  $T/T_{c0}$  ratio. The full line correspond to a value  $T/T_{c0} = 0.8$ , and the dashed line to  $T/T_{c0} = 0.9$ .

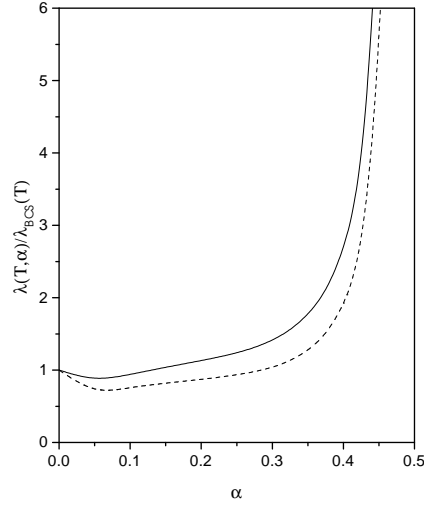


FIG. 6: The penetration depth ratio as function of the non-Fermi parameter  $\alpha$  for different values of the  $T/T_{c0}$  ratio. The full line correspond to a value  $T/T_{c0} = 0.8$ , and the dashed line to  $T/T_{c0} = 0.9$ .

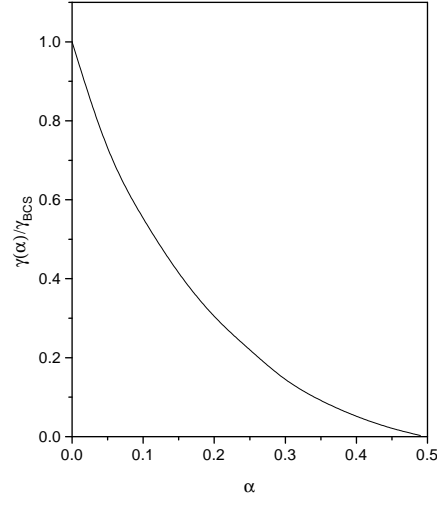


FIG. 7: The specific heat jump at the critical point ratio as function of the non-Fermi parameter  $\alpha$ .

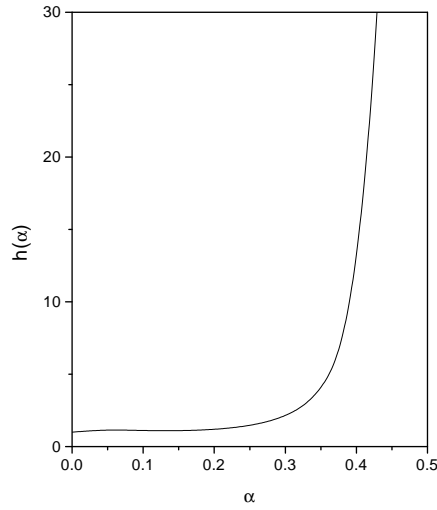


FIG. 8: The relative slope of the curve for the magnetic upper critical field as function of the non-Fermi parameter  $\alpha$ .